Chapter 5: Number Theory & The Real Number System

Math 120

Math for General Education

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Chapter 5:
Number Theory & the Real Number System

Number Theory – study of numbers and their properties.

- counting numbers or natural numbers
- begin with the number 1
- denoted by N = {1,2,3,4,5,…}

Prime & Composite Numbers

- Prime number is a natural number greater than 1 that has exactly two factors (or divisors), itself & 1.
  - 1st eight primes – 2,3,5,7,11,13,17,19
- Composite number is a natural number that is divisible by a number other than itself and 1.
  - 1st eight composites – 4,6,8,9,10,12,14,15

Rules of Divisibility

Divisible by	Test	Example
2	The number is even	924 is divisible by 2 since 4 is even
3	The sum of the digits is divisible by 3	924 is divisible by 3 (9+2+4 =15)
4	The last 2 digits are divisible by 4	924 is divisible by 4 (24 divisible by 4)
5	The number ends in 0 or 5	265 is divisible by 5
6	The number is divisible by both 2 and 3.	924 is divisible by 6; divisible by 2 & 3
8	The last 3 digits are divisible by 8	5824 is divisible by 8
9	The sum of digits is divisible by 9	837 is divisible by 9
10	The number ends in 0	290 is divisible by 10

10,368

2?Yes, it is even
3?Yes (sum of digits = 18)
4?Yes, 68 is divisible by 4
5?No
6?Yes, divisible by 2 & 3
8?Yes, 368 is divisible by 8
9?Yes, sum of digits = 18
10? No

Fundamental Theorem of Arithmetic

- Every composite number can be expressed as a unique product of prime numbers
- Two methods
  - Branching
  - Division
  - EX: 168

- Greatest common divisor (factor)
  - GCD (GCF)
  - Largest natural number that divides evenly every number in that set
- To find GCF of 2 or more numbers
  1. Determine prime factorization of each number
  2. Find each prime factor with smallest exponent that appears in each of prime factorizations
  3. Determine the product of the factors found in step 2.
- EX: GCF of 24, 48, 128

Least Common Multiple

- Least Common Multiple of a set of natural numbers is the smallest natural number that is divisible by each element of the set.
- Method not in the book - simpler
- EX: LCM of 24, 48, 128

Arithmetic & Geometric Sequences

- A sequence in which each term after the first term differs from the preceding term by a constant amount – arithmetic sequence
- Constant amount is called the common difference, d
- EX: 2,6,10,14,18,22,26,…
  - d = 4

Arithmetic & Geometric Sequences

- General or nth Term of an Arithmetic Sequence

$$a_n = a_1 + (n-1)d$$

EX: Find a_6 when $a_1 = 2$, $d = 3$

$$a_6 = a_1 + (6-1)d$$

$$a_6 = 2 + (5)3$$

$$a_6 = 17$$

Arithmetic & Geometric Sequences

- A *geometric sequence* is one in which the ratio of any term to the term that directly precedes it is a constant.
- The constant is called the common ratio, r
- EX: 2,4,8,16,32,64,128,…
  - r = 2

Arithmetic & Geometric Sequences

- General or nth Term of an Geometric Sequence

$$a_n = a_1 r^{n-1}$$

EX: Find a_6 when $a_1 = 3$, $r = 4$

$$a_6 = 3 \times 4^{(6-1)}$$

$$a_6 = 3 \times 4^5 = 3 \times 1024$$

$$a_6 = 3072$$

Fibonacci Sequence

- Named after Leonardo of Pisa
  - Also known as Fibonacci
- Fibonacci sequence

  1, 1, 2, 3, 5, 8, 13, 21, 34, …

  - 1st 2 terms are 1
  - Sum of these two terms is the third
  - Sum of 2nd & 3rd terms is 4th term, and so on.

Practice Problems

- Practice Problems
  - Pages 232-233
    - #18-56
  - Pages 294-296
    - #7-30
    - #39-62
    - #75, 77, 81, 83, 85
  - Pages 303-304
    - #23-30